A MATHEMATICAL MODEL FOR AUDIO CASSETTE PLAYING TIME

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INTRODUCTION

Most of us these days own, or have access to, an audio cassette recorder. Anyone who has done even a minimal amount of 'home' recording will appreciate that it is often useful to have some idea of the amount of time remaining at any particular stage of the record/playback process.

Cassette recorders are usually equipped with a digital tape counter which is either directly or indirectly connected to the take-up spindle. The usefulness of such a device is limited to providing an indication of where a particular track starts or finishes. In practice one would reset the counter to show 000 prior to the commencement of recording and at the completion of each track make a note of the counter reading.

This does not help us solve the problem of determining how much time remains. For example, we may initially set the counter to zero and find that by the time the tape has almost come to its end the counter reads 872. Suppose we have one more track to record and know that it is of 3½ minutes duration. Knowing that the counter will show 980, say, at the end of the tape is of no benefit to us in deciding whether or not we should proceed to record the last track.

Anyone who has studied the operation of a cassette recorder will have realised that the *linear* velocity of the tape past the record/playback heads must be constant. Since the radii of both spools is changing as tape is removed from one to the other, the rotation speeds must change so that constant linear velocity is preserved. The counter is directly connected to the take-up reel, and as more and more tape is transferred to that reel, the counter speed becomes gradually slower.

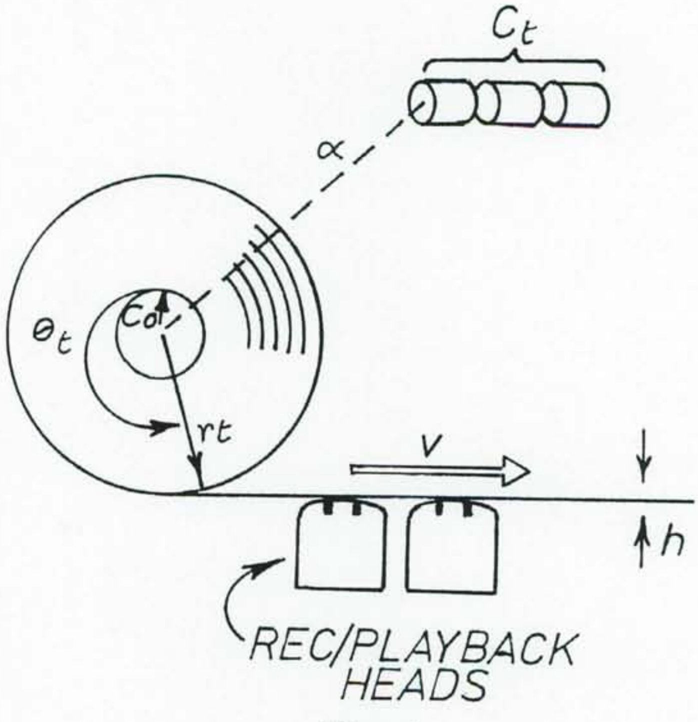
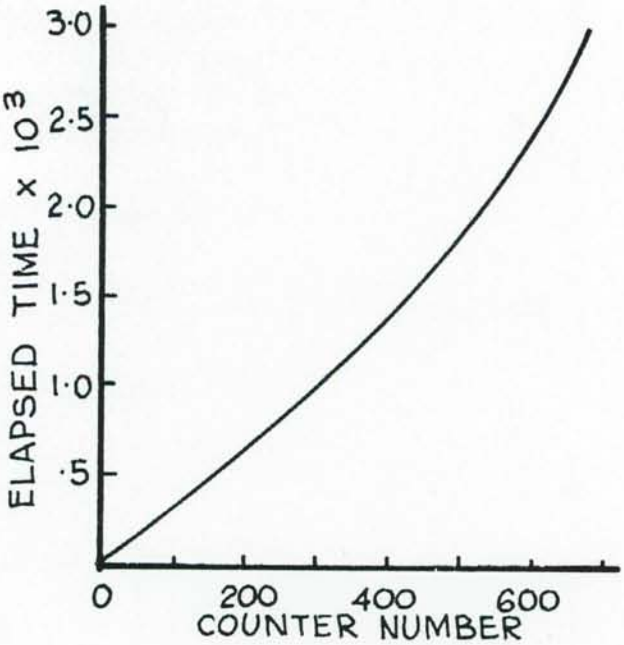


Fig. 1

A plot of elapsed time versus counter number is shown in figure 2. It is evident that this curve is not linear, as agreed above.



In order to be more accurate than the graphical approach permits, it is necessary to model the situation mathematically.

The first approach is theoretical in its treatment and perhaps would be in keeping with the way an Applied Mathematician would approach the problem.

An alternative, empirical, approach has been included which might correspond with the pursuits of one who doesn't want to get involved "in modelling the system" per se, but who requires a quick, readily obtainable rule based on observation and collection of data from the system he or she is attempting to describe.

It will be seen that this latter approach will nevertheless provide a useful check on the adequacy of the first approach. It is noted here that the author had modelled the system several years ago and has been happily (and successfully) applying his results since – with the aid of a computer print-out.

A MATHEMATICAL MODEL

The essential components of the system to be described are shown in figure 1.

The variables requiring definition here are:

- r_o – initial radius of spool (without any tape)
- θ_t – angular displacement of spool at time t
- r_t – radius of spool at some time t
- h – tape thickness
- n_t – number of revolutions of spool after time t
- α – constant of proportionality relating spindle revolutions to counter revolutions
- c_t – digital counter number after time t ($c_t = \alpha n_t$)

We have immediately,

$$r_t = r_o + n_t h \qquad (1)$$

also

$$n_t = \frac{\theta_t}{2\pi} \qquad (2)$$

and hence

$$r_t = r_o + \frac{\theta_t}{2\pi} h \qquad (3)$$

Now if the tape has constant linear speed, V, say then

$$r\omega_t = V$$

where ω_t is the angular velocity at same time t

$$\Rightarrow \omega_t = \frac{V}{r}$$

and by (3) we have:

$$\frac{d\theta}{dt} = \frac{V}{r_o + \frac{h}{2\pi}\theta}$$

Separating variables of this general first order d.e. yields

$$\int_o^\theta (r_o + \frac{h}{2\pi}\theta), d\theta = \int_o^t V dt$$

$$\Rightarrow r_o\theta + \frac{h}{4\pi}\theta^2 = Vt$$

and

$$t = \frac{r_o}{V}\theta + \frac{h}{4\pi V}\theta^2$$

and by (2) we obtain:

$$t = \frac{\alpha \pi r_o}{V} n + \frac{\pi h}{V} n^2$$

and since $c = \alpha n$

$$t = \frac{\alpha \pi r_o c}{\alpha} + \frac{\pi h c^2}{\alpha^2} \qquad (4)$$

Obviously, in order to use equation (4) we need to obtain values for the parameters r_o, h, and α.

A crude measurement on my equipment gave $r_o = 2.15$ cm.

One must be careful when establishing a value for h since this will vary depending on the playing time of the cassette used (many will have experienced the problems caused by the very thin tape used in C-120 cassettes!)

Industry sources suggest the following values for h for different cassette types:

Cassette Type	h (mm)
C-30	0.0165
C-60	0.0165
C-90	0.0122
C-120	N/A

Likewise α will have to be determined by measurement for your own machine. One simple way of doing this is to observe the operation of the player with no tape in place and take repeated measurements on the ratio of counter number to number of spindle revolutions. This ratio should be relatively constant and is therefore the correct value to use for α. (For my equipment I obtained $\alpha = 0.575$).

Having determined these values you may be then interested in establishing the validity of the model by comparing predicted times with actually observed values. You may find after doing so that there is perhaps unacceptable discrepancy.

Apart from inherent sources of error in the measurement of r_o, α, and to a lesser extent, h, another problem which arises which has not been accounted for by equation (4) concerns the packing of successive layers of tape.

Reference

1. KALMAN, D., *A Model for Playing Time,* Mathematics Magazine, Vol. 54, No. 5, November 1981.

BOOK REVIEWS (cont.)

problems are followed toward solution to illustrate this, demonstrating the kinds of problems which have solutions computers can assist with. These examples introduce ideas such as flowcharts as incomplete representations of solutions, decision points, looping and the blurred distinction between data and instructions.

Later sections examine hardware components and the principles behind their functioning. An introduction to programming in BASIC is provided. Applications of computers in everyday life and industry are examined more closely. These sections further clarify what computers can do, and given that some problems are solved by people, what computers may be able to do.

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LETTERS (cont.)

specifically includes accounting, chemistry, computer science, environmental science, physical science and physics. A further rule states "a candidate will not be granted access to mains electricity supply within a examination room." I would like to ask at what level are you teaching where sixth formers are not allowed to use computers in accounting? physics? computer science?

There is an answer of course. The Geelong Church of England Grammar School has just placed an ad in Computer Weekly for a teacher in Computer Studies with "experience in introducing computer work across the curriculum" to start in Term 1, 1983. Perhaps the fact that Geelong Grammar finds VISE irrelevant can give hope for the future.

Yours faithfully,

Dr. Brenton R. Groves