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A Hospital Throughput Model in the Context of Long Waiting Lists

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In a public health system, one of the problems is the size of the waiting list for admission to hospital. This research involves establishing a method of analysing the general surgery waiting list problem at hospital and district level. While there are many aspects to such a study, this paper concentrates on a linear programming model to plan the aggregate throughput of the general surgical department. Preliminary results from applying the techniques to actual health districts in the United Kingdom are reported.

INTRODUCTION

MOST health authorities in the National Health Service (N.H.S.) in the United Kingdom are concerned about the length of their waiting lists, and it is a normal criticism of the health service that waiting time for treatment is excessive. The South Western Regional Health Authority requested a study into the waiting list situation in the Region. The brief was very broad and required the development of a better understanding of the problem and a means of analysing that would aid in reducing waiting times.

The size of the waiting list is the result of the demand for treatment and the rate at which patients can be treated. Where demand is greater than throughput, a queue is inevitable. In reality the queue is kept to a finite size by the long waiting time, deterring general practitioners from referring patients, and consultants from admitting patients to the waiting list. Thus the policy of a particular consultant can alter the waiting list considerably. Furthermore it has been observed that waiting lists continue to exist and even maintain their size subsequent to an improvement in hospital throughput (see Culyer and Cullis[1] and Snaith[2]).

Considerable work has been done describing the waiting list problem. Much of this is in internal papers within the health service, but there is published work, e.g. Luck *et al.*,[3] Curnow,[4] Butterly,[5] Baderman *et al.*[6] and Frost.[7] Included in this work is research on the improvement of referral of patients to consultants and other questions of waiting list management. While a smoother running waiting list system may be an advantage, it has also a tendency to increase the waiting list because there is no corresponding increase in hospital throughput. We have approached the problem from the point of view of the throughput because this is the aspect of the system that is ultimately the bottleneck.

Factors affecting the supply of patients relate to the population structure and expectation as well as the referral policies of the consultants and general practitioners. The rate of throughput is affected by the quantity of resources available, discharge and admission policies, ward and theatre scheduling. A more detailed statement of these factors is in Fox and Canvin.[8] George *et al.*[9] discuss questions of how to measure the severity of a waiting list and other aspects of this study from a medical and waiting list viewpoint.

PATIENT THROUGHPUT MODEL

It is not a new idea that waiting lists can be improved by increasing throughput. Many studies have been done to increase throughput. They have, however, concentrated on day-to-day management problems. Luckman *et al.*[10] developed a simulation model to

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assess the viability of large wards. This model is also applicable for ward management, which is demonstrated in Luckman.[11] The latter study involves the planning of a surgical unit. Butterly[5] also develops a series of simulation models to assess waiting-list policies and admission/discharge policies. Hospital bed usage has been researched by other authors. Morris and Handyside[12] compared the effect on bed occupancy of various emergency admission policies. Newell[13] discusses aspects of hospital practice that lead to inefficient use of beds. A completely different approach by Loudon[14] is to examine alternatives to hospital admission. The current increase in day surgery is a reflection of this emphasis. In all, these studies attempt to increase throughput by reducing the length of a patient's stay in hospital, by increasing the percentage bed occupancy or by eliminating hospital admission completely. Our emphasis is the broader issue of overall resource balance and usage. This approach complements rather than replaces the more detailed analyses.

Since the largest number of patients and the longest waiting lists are for general surgery, this study has concentrated (although not exclusively) on this speciality. Clearly, the model proposed is appropriate for other specialities or even applications in a framework other than reducing waiting lists. Indeed in the district investigated, the study became one of rescheduling resources among specialities. Most previous planning at an aggregate resource level has ignored any breakdown into patient groups. Such a disaggregation is essential for this study and constitutes the kernel of the analysis. Figure 1 gives a diagrammatic representation of the model.

PURPOSE OF THE MODEL

Within the scope of the whole study, the aim of this linear programme is to model a surgical department (in this case general surgery) of a district or hospital. In particular it is aimed towards answering questions about the optimal throughput of patients, taking account of their urgency, diagnosis and resource use and the availability of resources. Also it looks at the effect on throughput of changes such as additional resources, a reallocation of resources or more efficient use of resources.

DECISION VARIABLES

In modelling the system, the crucial problem is how to classify admissions so as to retain the characteristics (in terms of resource use) of different diagnoses, different levels of urgency (i.e. emergencies, urgent planned admissions, routine planned admissions) and different forms of treatment (i.e. inpatient admission and day surgery). There is the additional complication that patients in the same diagnostic category may receive different operations or perhaps no operation. Emergencies have a much higher probability of not having an operation than planned admissions. (For example in the district we have studied, 45% of emergencies have operations compared with 93% of planned admissions.)

Diagnostic category is used as the main classification of patients. Since there are many hundreds of separate diagnoses, careful consideration has been given to the aggregation of these. The criteria applied to this were:

(a) similarity of the medical nature of the diagnoses;
(b) number of cases admitted of the diagnoses;
(c) similarity of types of operations performed on the diagnoses;
(d) significance of the diagnosis to admission decisions when marginal changes to the resources are made (e.g. a diagnosis that occurs almost exclusively among emergencies will not be affected by a marginal change in resources).

The 37 categories used are listed in Appendix A.

The primary decision variables are the number of admissions of each aggregate diagnostic category broken down by level of urgency and patient type (inpatient/day patient). Level of urgency is considered because, for a given diagnosis, the condition can be of varying severity. The model gives priority to urgent admissions over routine ones. Direct

TABLE 1. URGENCY LEVELS OF GENERAL SURGICAL PATIENTS

Urgency level	Type and source of admissions	Objective function weight
1	Emergency	100
2	Booked and waiting list ($\leqslant$30 days)	100
3	Waiting list (1–6 months)	4
4	Waiting list ($\frac{1}{2}$–2 years)	2
5	Waiting list ($\geqslant$ 2 years)	1

data on the number of admissions at varying levels of urgency are not readily available. The length of time spent by patients on the waiting list is used as a surrogate measure of urgency. This, of course, varies widely between districts. For the district we studied the urgency classifications are given in Table 1.

Secondary variables have been included that give the number of operations performed. Like diagnoses, there are hundreds of different operations performed in the general surgery department. For the purposes of this model, it is the length of time an operation takes that is important, since that determines the amount of theatre, consultant and nursing resources used. The operations have been aggregated into categories according to their length of time in the theatre. The categories are (in minutes) 15–30, 30–60, 60–90, 90–120, 120–150, 150–180 and over 180. An operation is classified according to the expected time in theatre. These times were given by a surgeon and verified from the theatre book.

OBJECTIVE FUNCTION

The objective of the model is to find the optimal throughput of patients, giving preference to the categories with higher urgency. To do this we weight the variables with greater weights for higher urgency. Different diagnoses may also be given different weights according to their priority of admission. Since these weights are largely subjective, deciding their value is an iterative process. If a solution has undesirable characteristics because of the weights used, a new solution can be found by altering the weights in line with the decision maker's judgement. This illustrates the exploratory nature of the use made of the model, even though on the surface it is an optimizing model. Initially, the values of the weights were arbitrarily chosen to fulfil the urgency criterion. They are given in Table 1.

CONSTRAINTS

The constraints are:

(a) The number of admissions in each diagnostic/urgency category

These are based on historical data for the categories. For those diagnoses where it can be assumed that all known cases are treated, no additional admissions are permitted. Additional admissions are restricted to the diagnoses that represent the reservoir of patients who have not been treated because of insufficient hospital resources.

(b) The available bed space measured in bed-days

The mean length of stay for each diagnostic/urgency category is available. The bed-days available is derived from the number of general surgical beds corrected for their mean occupancy level.

(c) Consultant surgeons' hours

The operating time and ward round time are the significant components of the consul-

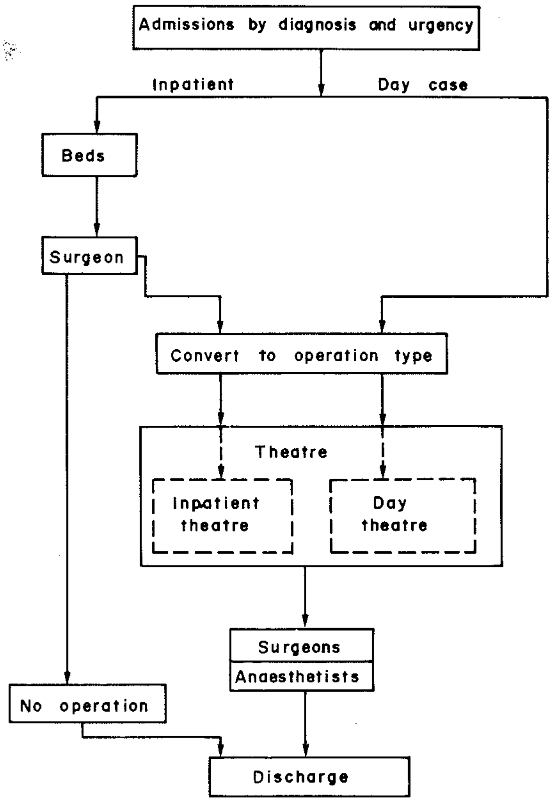

FIG. 1. *Flow chart of throughput model.*

tants' time. These times correspond to the actual schedules of the surgeons. Time spent in outpatients' clinic is deducted from consultants' time before that figure is entered into the analysis. Registrars are included in so far as they supplement the consultants' time.

(*d*) *The availability of theatre space measured in allocated theatre-operating-hours*

Data on operating times is assumed in the definition of the operation types. Theatre time available takes into account emergency theatre sessions, built in spare capacity for urgent operations and time required to maintain the theatres.

(*e*) *The definitional equations for the operation variables*

For each admission diagnostic/urgency category there is a distribution of operation types. The number of operations performed is generated using this distribution. Historical data is used to find the values of the coefficients in these relationships.

Appendix A gives the algebraic structure of the model.

RESULTS

Preliminary results were obtained for the general surgery departments of two hospitals in one health district. These are serviced by the same surgeons and are close enough geographically to permit some interchange of patients. The first hospital is the District General Hospital (D.G.H.) and the other a local hospital with good theatre facilities but no intensive care unit.

Mere observation showed the probable resource bottlenecks at the hospitals. The D.G.H. had unused theatre sessions yet an apparently serious problem in admitting patients within a reasonable time. Calculations revealed that in 1979 the general surgery department was, in effect, using 11 beds beyond its allocation. Bed space seemed to be the bottleneck. In contrast the local hospital had all its theatre sessions booked. However, even though the general surgical beds were fully used, overall the acute beds had a very low occupancy rate. In the light of this a series of runs were made for each of the hospitals.

D.G.H. run 1—verification run

The first run, using the actual 1979 admissions, was used to verify the accuracy of the model. It also gave information on the degree of slack in the plentiful resources. Table 2 shows that the model estimates that approximately the same number of beds are required. The mean inpatient operating time per operation of 70 minutes corresponds closely with a time of 72 minutes, as sampled from the theatre book. Of interest is the slack in surgeon time and theatre time. In particular, the theatres are utilized for actual operating for only 70% of the time booked. Whether this is an acceptable utilization rate and, if not, how it could be improved are open questions. Day theatre utilization is even lower, and here no bed constraint is applicable.

D.G.H. run 2—increased beds

Bed availability can, in theory, be increased either by allocating more beds to general surgery or by reducing the mean length of stay of patients. Consider an increase in available bed days of 10%. To use these beds and some of the under-utilized day theatre, we allowed a maximum increase in annual intake, over the 1979 figures, of 100 patients for each of hernia, gall bladder and prostate (inpatients) and hernia and varicose veins (day patients). Table 2 shows that all of these extra patients could be treated without using all the extra beds. In fact, if theatre utilization could be increased to 81%, then no extra theatre sessions were required either. Since routine planned admissions are the heart of the waiting list, it is interesting to observe that they could be increased by 71% by increasing the beds available—or reducing length of stay—by less than 10%.

TABLE 2. RESULTS OF GENERAL SURGERY RUNS—D.G.H. 1979

Resources	Run 1 Available	Run 1 Calculated usage	Run 2 Available	Run 2 Calculated usage	Run 3 Available	Run 3 Calculated usage
Beds @ 82% occup.	103	102	114	111	92	92
Theatre hours						
Inpatients	4000	2820	4000	3200	4000	2380
Day cases	1000	350	1000	460	1000	460
Surgeon hours	6000	4200	6000	4780	6000	3770
Anaesthetist hours	5000	3160	5000	3660	5000	2840
Inpatients						
Emergencies	2610		2610		2610	
Urgent planned	740		740		740	
Routine planned	420		720		10	
Total	3770		4070		3360	
Day patients	790		990		990	
Theatre procedures						
Inpatients	2430		2730		2030	
Day patients	770		970		970	
Proportion of emergencies to inpatients	69%		64%		78%	
Theatre utilisation						
Inpatient	71%		81%		60%	
Day	34%		45%		45%	
Weekly surgeon hours	98 hours		111 hours		88 hours	
Average theatre time per procedure						
Inpatient	70 min		70 min		70 min	
Day	27 min		28 min		28 min	

D.G.H. run 3—beds reduced to actual allocations

What would be the effect of restricting general surgery to its actual bed allocation and not allowing it access to the extra 10 beds it was using? If this option was adopted, the results in Table 2 show that routine planned admissions almost cease. Only 10 can be made. The conclusion is that the viability of the general surgical department depends on its regularly overspilling its allocation. Clearly, a reallocation of beds is required.

Local hospital

Having verified the data for the local hospital, we posed the question "What quantity of resources would be required at the local hospital to treat the 1979 routine planned admissions of both that hospital and the D.G.H.?" This involves running the model with no limits on the resources—only the upper bounds on the admissions.

The result was that 12 beds above the current general surgical allocation would be needed. These are available. In the theatre an extra two general surgical sessions—assuming 80% utilization—are required. These are not immediately available but could be made available if a compromise with another speciality could be achieved.

Since the model is not designed as a multi-hospital model, further calculations were made that involved altering the mix of patients between the hospitals. There may be residential and medical reasons why some patients cannot be transferred. Table 3 summarises the results. With no increase in either theatre sessions at the local hospital or beds available at the D.G.H., a shift of 'long stay' patients from the D.G.H. to the local hospital and 'short stay' patients from the local hospital to the D.G.H. achieves a large increase in routine planned admission. Clearly, more beds are needed at the local hospital and more theatre time at the D.G.H. Both of these resources are plentiful.

The use of subjective weights in the objective function is a risky procedure since the choice of weights can significantly affect the results. Fortunately, in this study they did not pose an important problem. In all runs performed, the weights were set so as to ensure that the emergency and urgent planned patients received priority over routine planned admissions. In fact, in all the runs all urgent patients were admitted. When resources are insufficient to admit all routine patients, those admitted were chosen by the model according to their use of the scarce resources. In that case the relative weights of different diagnostic categories and different levels of urgency within the routine admissions affected the choice of categories admitted. Strictly, it is at this stage that medical staff have to interact with the solution to ensure that the weights give medically sensible results. In fact, for the runs performed, a change in weights for the different levels of routine admissions have almost no effect because of the nature of the runs. Tests were made for the sensitivity of the solutions to the weights. The effect of giving urgency levels

Table 3. Reallocation between D.G.H. and local hospital

Details	Routine planned patients	Urgent planned patients
Patients transferred from D.G.H. to local hospital	200 long stay (11 days average)	230 long stay (16 days average)
Patients transferred from local hospital to DGH	200 short stay (3 days average)	230 short stay (6 days average)
Additional beds required at local hospital	5*	7.5*
Additional routine admissions possible at D.G.H. as a result of the transfer	360 short stay (4 days) or 140 long stay (11 days) or 100 short stay plus 100 long stay	520 short stay or 200 long stay or 140 short stay plus 140 long stay

* Readily available.

3, 4 and 5 the same weight was that the low bed-occupancy diagnoses received preference over the high bed-occupancy ones, irrespective of urgency. It is for the decision maker to assess such tradeoffs and find acceptable weights. In any event it us unlikely that a situation will occur where emergency and urgent planned patients are not admitted. So the effect of the weights will always be limited.

ROLE OF THE MODEL IN THE TOTAL PROJECT

The model was not used to find an optimal solution for direct and complete implementation. Rather it is an indicative tool, pointing out the direction of likely improvements and giving broad measures of the impact of these changes on the throughput of patients. Its function was as much calculating the resource requirements of various policy options as providing an optimum.

As usual, a lot of the benefit in this study was gained at the model construction stage. The resource use of different patient categories was particularly insightful. This highlighted both the different resource requirements of a hospital receiving emergency admissions over a hospital receiving only planned admissions and the resources required to treat additional waiting list patients (i.e., the marginal resources as opposed to the average resources).

Since the model was designed to answer problems about physical throughput, it was unnecessary to build in costs. For the runs performed, costs were almost irrelevant. No significant additional expenditure was possible, so it was purely a matter of the more efficient use of existing resources. In other situations cost calculations can be performed either by introducing an additional constraint or by calculations on the side. The omission of costs from the model does not imply that they are unimportant—merely that the model is only a part of the total analysis.

Subsequent to the study of the general surgery department, work has begun on the resource requirements of all the departments in the hospital. Much of this analysis will not require a model of the form described. However, it is being adapted to gynaecology and other surgical departments.

CONCLUSIONS

The hospital throughput model was derived to aid in the reduction of surgical waiting lists. While only a part of the total analysis, this tool for finding the optimal aggregate throughput has become of interest in its own right. In particular it is of use in assessing the changes in throughput for many proposed alterations to the hospital resource allocation, e.g. changes in theatre allocation, bed allocation, surgeon time, etc.

Since it is an aggregated model, its results should be used to get a broad view of the hospital throughput and an indication of the types of patients receiving preference (if of equivalent status medically) rather than to give precise numbers of patients to be admitted.

APPENDIX A: GENERAL SURGERY DIAGNOSTIC CLASSIFICATIONS

1.0 Factors influencing health status and contact with health services (other)
1.1 Sterilisation
1.2 Procedures not carried out
2.0 Infectious and parasitic diseases
3.0 Malignant neoplasms (other)
3.1 —stomach, oesophagus, small intestine, colon, rectum and anus
3.2 —female breast
3.3 —prostate
3.4 —bladder

4.0 Other neoplasms (other)
4.1 Benign neoplasms
4.2 Neoplasms of uncertain behaviour
5.0 Diseases of the nervous, circulatory and respiratory system (other)
5.1 Varicose veins
5.2 Haemorrhoids
6.0 Diseases of the digestive system (other)
6.1 Oesophagus
6.2 Stomach and duodenum (ulcers, gastritis etc)
6.3 Appendicitis
6.4 Hernia
6.5 Intestine
6.6 Anal fissure and fistula
6.7 Gall bladder
7.0 Diseases of genito-urinary system (other)
7.1 Lower urinary tract and bladder
7.2 Urethra and urinary tract
7.3 Prostate
7.4 Male genital organs
7.5 Breast
8.0 Diseases and disorders—other parts (other)
8.1 Pilonidal cyst
8.2 Diseases, inflammatory conditions and infections of skin and tissue
8.3 Congenital abnormalities of genital organs
9.0 Ill-defined conditions symptoms and signs (other)
9.1 Abdomen
10.0 Injury and poisoning
10.1 Intracranial injury

APPENDIX B: ALGEBRAIC FORMULATION

Variables

x_{ij}^I—number inpatient admissions diagnosis i urgency j, (i,j).
x_{ij}^D—number day patient admissions diagnosis i urgency j, (i,j).
y_k^I—number operations type k (inpatients).
y_k^D—number operations type k (day patients).

Constants

A_{ij}^I, A_{ij}^D—upper bound of inpatients (i,j)/day patients (i,j).
b_{ij}—length of stay of inpatients diagnosis i urgency j.
d_{ijk}^I, d_{ijk}^D—proportion of operations type k performed on inpatients (i,j)/day patients.
t_k—average theatre time of operation type k.
c_{ij}—consultant time (not operating) on inpatient (i,j).
C—consultant time.
T^I, T^D—theatre time inpatients/day patients.
B—bed space in bed days.
w_i—priority weighting for diagnosis i.
r_j—priority weighting for urgency j.

Constraints

Admissions

$$x_{ij}^I \leqslant A_{ij}^I; \quad x_{ij}^D \leqslant A_{ij}^D \quad \text{all } i, j.$$

Beds

$$\sum_{i,j} b_{ij} x_{ij}^I \leqslant B.$$

Conversion to operations

$$\sum_{i,j} d^I_{ijk} x^I_{ij} = y^I_k, \quad \text{all } k; \quad \sum_{i,j} d^D_{ijk} x^D_{ij} = y^D_k, \quad \text{all } k.$$

Theatre time

$$\sum_k t_k y^I_k \leqslant T^I; \quad \sum_k t_k y^D_k \leqslant T^D.$$

Surgeon time

$$\sum_{i,j} c_{ij} x^I_{ij} + \sum_k t_k (y^I_k + y^D_k) \leqslant C.$$

Objective

$$\text{Maximize} \quad \sum_i w_i \sum_j r_j (x^I_{ij} + x^D_i).$$

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